Temperature-dependent equation of state of condensed matter

This article has been downloaded from IOPscience. Please scroll down to see the full text article.
1997 J. Phys.: Condens. Matter 92987
(http://iopscience.iop.org/0953-8984/9/14/013)
View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 171.66.16.207
The article was downloaded on 14/05/2010 at 05:38

Please note that terms and conditions apply.

# Temperature-dependent equation of state of condensed matter 

Piyush Kuchhal, Ravindra Kumar and Narsingh Dass<br>Physics Department, University of Roorkee, Roorkee-247667, India

Received 18 September 1996, in final form 18 December 1996


#### Abstract

In the present paper, a temperature-dependent equation of state (EOS) of condensed matter is discussed, which is capable of predicting the high-pressure and high-temperature behaviour of solids and liquids. The EOS can be used to obtain the volume compression, the isothermal bulk modulus and its first pressure derivative, the thermal expansion coefficient and the Anderson-Grüneisen parameter, together with other thermodynamic properties as functions of pressure at different temperatures. The present EOS has been applied for $\mathrm{Au}, \mathrm{Mo}$ and W and liquid Hg . Good agreement between theory and experiment is observed.


## 1. Introduction

The equation of state (EOS) of condensed matter ( $P-V-T$ relation) is very important in many fields of basic and applied sciences including physics and geophysics. However, many different forms of isothermal EOS ( $P-V$ relation) are available in the literature [1-4] but have been found to have limited uses as far as the practical applications are concerned. Hence, there is a need for a temperature-dependent EOS which will have a wide scope in practical applications in physics and geophysics.

On the one hand, we find the temperature-dependent EOS given by Vinet et al [5] and another given by Parsafar and Mason [6]. On the other hand, Kumari and Dass [4] have suggested an EOS which is applicable to a large class of solids [7] and liquids [8]. Therefore, it will be of much interest to see the validity and the usefulness of this EOS after adding thermal effects to make it a temperature-dependent EOS. Hence, the aim of the present paper is to suggest a temperature-dependent EOS which is successful in the high-pressure and high-temperature regions.

## 2. Theory

The EOS given by Kumari and Dass [4] can alternatively be written at a reference temperature $T_{R}$ as

$$
\begin{equation*}
V\left(P, T_{R}\right) / V\left(0, T_{R}\right)=[(1+\beta) \exp (Z P)-\beta]^{-1 / \eta} \tag{1}
\end{equation*}
$$

The other relations obtained during the development of the EOS are

$$
\begin{equation*}
B_{T}\left(P, T_{R}\right)=B_{T}\left(0, T_{R}\right)+\frac{B_{T}^{\prime}\left(0, T_{R}\right)}{Z}[1-\exp (-Z P)] \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
B_{T}^{\prime}\left(P, T_{R}\right)=B_{T}^{\prime}\left(0, T_{R}\right) \exp (-Z P) \tag{3}
\end{equation*}
$$

In equations (1)-(3), $B_{T}^{\prime}\left(0, T_{R}\right)$ is the first pressure derivative of the isothermal bulk modulus $B_{T}\left(P, T_{R}\right)$ at $P=0, Z$ is a pressure-independent parameter, $\beta=$ $B_{T}^{\prime}\left(0, T_{R}\right) /\left[B_{T}\left(0, T_{R}\right) Z\right]$ and $\eta=B_{T}^{\prime}\left(0, T_{R}\right)+B_{T}\left(0, T_{R}\right) Z$.

To include the thermal effects into the EOS, we use the simplest form

$$
\begin{equation*}
P(T)=P\left(T_{R}\right)+\alpha\left(0, T_{R}\right) B_{T}\left(0, T_{R}\right)\left(T-T_{R}\right) \tag{4}
\end{equation*}
$$

where $\alpha\left(0, T_{R}\right)$ is the thermal expansion coefficient. Equation (4) has been used extensively (see, e.g., $[5,6,9]$ ).

Putting equation (4) into equations (1)-(3), we get

$$
\begin{align*}
& V(P, T) / V\left(0, T_{R}\right)=[(1+\beta) \exp \sigma-\beta]^{-1 / \eta}  \tag{5}\\
& B_{T}(P, T)=B_{T}\left(0, T_{R}\right)+\frac{B_{T}^{\prime}\left(0, T_{R}\right)}{Z}[1-\exp (-\sigma)] \tag{6}
\end{align*}
$$

and

$$
\begin{equation*}
B_{T}^{\prime}(P, T)=B_{T}^{\prime}\left(0, T_{R}\right) \exp (-\sigma) \tag{7}
\end{equation*}
$$

where $\sigma$ is taken as

$$
\sigma=Z\left[P-\alpha\left(0, T_{R}\right) B_{T}\left(0, T_{R}\right)\left(T-T_{R}\right)\right]
$$

under the condition that $Z$ is also a temperature-independent parameter.
Thus, equation (5) represents a temperature-dependent EOS of condensed matter and will be tested in solids and liquids to determine its validity at high pressures and high temperatures.

The following important results can be obtained from equations (5)-(7).
(i) Equation (5) can also be written as
$P=\frac{1}{Z} \ln \left(\frac{\left(V(P, T) / V\left(0, T_{R}\right)\right)^{-\eta}+\beta}{1+\beta}\right)+\alpha\left(0, T_{R}\right) B_{T}\left(0, T_{R}\right)\left(T-T_{R}\right)$.
Thus, the main advantage of the present EOS given by equation (5) is that it is an inverted EOS, i.e. we can write the volume as a function of pressure and temperature ( $V=f(P, T)$ ) given by equation (5) or the pressure as a function of volume and temperature $(P=f(V, T))$ given by equation (8). Hence, the present EOS definitely has the advantage over all those EOSs which are non-invertible but can be expressed in many forms.
(ii) Together with $V(P, T)$, we can also study simultaneously $B_{T}(P, T)$ and $B_{T}^{\prime}(P, T)$ as functions of both the pressure and the temperature with the help of equations (6) and (7), respectively.
(iii) Differentiation of equation (8) with respect to temperature at constant volume taking $Z$ as the temperature-independent parameter gives

$$
\begin{equation*}
\left(\frac{\partial P}{\partial T}\right)_{V}=\alpha(P, T) B_{T}(P, T)=\alpha\left(0, T_{R}\right) B_{T}\left(0, T_{R}\right) \tag{9}
\end{equation*}
$$

Equation (9) represents the same results which have already been obtained by Kumari and Dass [10] from quite a different approach. Many interesting results can be obtained with the help of equation (9) and a few are listed here.
(a) Both the thermal expansion coefficient $\alpha(P, T)$, and the Anderson-Grüneisen parameter $\delta_{T}(P, T)$ can be studied as functions of pressure at different temperatures [11].
(b) An isobaric EOS can be obtained [10].
(c) The pressure dependence of the Grüneisen parameter $\gamma(P, T)$ can be studied [12].

Table 1. Input data ( $10 \mathrm{kbar}=1 \mathrm{GPa}$ ).

| Solids | Pressure range <br> $(\mathrm{kbar})$ | Temperature range <br> $(\mathrm{K})$ | $T_{R}$ <br> $(\mathrm{~K})$ | $B_{T}\left(0, T_{R}\right)$ <br> $(\mathrm{kbar})$ | $B_{T}^{\prime}\left(0, T_{R}\right)$ | $Z$ <br> $\left(\times 10^{-4} \mathrm{kbar}^{-1}\right)$ | $\alpha\left(0, T_{R}\right)$ <br> $\left(\times 10^{-6} \mathrm{~K}^{-1}\right)$ | RMSD at <br> $T_{R}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Au | $0.0-2161.0$ | $300.0-3000.0$ | 300.0 | 1691.52 | 5.04 | 16.09 | $42.78[17]$ | $1.54 \times 10^{-4}$ |
| Mo | $0.0-3000.0$ | $293.0-7605.0$ | 293.0 | 2709.02 | 3.58 | 0.84 | $15.00[17]$ | $2.67 \times 10^{-3}$ |
| W | $0.0-3000.0$ | $290.0-6905.0$ | 293.0 | 3138.04 | 3.68 | 0.90 | $13.5[17]$ | $4.05 \times 10^{-3}$ |
| Hg | $0.0-14.0$ | $295.05-423.15$ | 295.05 | 248.44 | 9.17 | 158.1 | $181.607[15]$ | $2.04 \times 10^{-5}$ |

Table 2. Comparison of pressure (GPa) as a function of volume and temperature in Au: $P_{\text {exp }}$, experimental values taken from Heinz and Jeanolz [13]; $P_{\text {calc }}$, our calculated values.

| $V / V_{0}$ | $T=300 \mathrm{~K}$ |  | $T=500 \mathrm{~K}$ |  | $T=1000 \mathrm{~K}$ |  | $T=1500 \mathrm{~K}$ |  | $T=2000 \mathrm{~K}$ |  | $T=2500 \mathrm{~K}$ |  | $T=3000 \mathrm{~K}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $P_{\text {exp }}$ | $P_{\text {calc }}$ | $P_{\text {exp }}$ | $P_{\text {calc }}$ | $P_{\text {exp }}$ | $P_{\text {calc }}$ | $P_{\text {exp }}$ | $P_{\text {calc }}$ | $P_{\text {exp }}$ | $P_{\text {calc }}$ | $P_{\text {exp }}$ | $P_{\text {calc }}$ | $P_{\text {exp }}$ | $P_{\text {calc }}$ |
| 1.000 | 0.00 | 0.00 | 1.43 | 1.45 | 5.02 | 5.06 | 8.62 | 8.68 | 12.22 | 12.30 | 15.82 | 15.92 | 19.42 | 19.54 |
| 0.980 | 3.56 | 3.60 | 4.96 | 5.04 | 8.50 | 8.66 | 12.05 | 12.28 | 15.60 | 15.90 | 19.15 | 19.52 | 22.70 | 23.13 |
| 0.960 | 7.61 | 7.66 | 8.99 | 9.11 | 12.48 | 12.73 | 15.98 | 16.35 | 19.47 | 19.96 | 22.97 | 23.58 | 26.47 | 27.20 |
| 0.940 | 12.22 | 12.27 | 13.58 | 13.71 | 17.01 | 17.34 | 20.46 | 20.95 | 23.90 | 24.57 | 27.35 | 28.19 | 30.80 | 31.81 |
| 0.920 | 17.46 | 17.50 | 18.80 | 18.95 | 22.18 | 22.56 | 25.58 | 26.18 | 28.97 | 29.80 | 32.37 | 33.42 | 35.76 | 37.04 |
| 0.900 | 23.44 | 23.44 | 24.75 | 24.89 | 28.08 | 28.51 | 31.42 | 32.13 | 34.77 | 35.74 | 38.11 | 39.36 | 41.45 | 42.98 |
| 0.880 | 30.24 | 30.21 | 31.54 | 31.66 | 34.81 | 35.28 | 38.10 | 38.90 | 41.39 | 42.51 | 44.68 | 46.13 | 47.97 | 49.75 |
| 0.860 | 38.00 | 37.94 | 39.27 | 39.38 | 42.49 | 43.00 | 45.73 | 46.62 | 48.97 | 50.24 | 52.20 | 53.86 | 55.44 | 57.47 |
| 0.840 | 46.85 | 46.77 | 48.10 | 48.21 | 51.27 | 51.83 | 54.45 | 55.45 | 57.63 | 59.07 | 60.82 | 62.69 | 64.01 | 66.30 |
| 0.820 | 56.97 | 56.87 | 58.19 | 58.32 | 61.31 | 61.94 | 64.43 | 65.56 | 67.56 | 69.17 | 70.70 | 72.79 | 73.83 | 76.41 |
| 0.800 | 68.54 | 68.46 | 69.74 | 69.91 | 72.80 | 73.52 | 75.87 | 77.14 | 78.95 | 80.76 | 82.03 | 84.38 | 85.11 | 87.99 |
| 0.780 | 81.79 | 81.76 | 82.97 | 83.21 | 85.97 | 86.82 | 88.99 | 90.44 | 92.02 | 94.06 | 95.04 | 97.68 | 98.07 | 101.30 |
| 0.760 | 97.00 | 97.04 | 98.16 | 98.50 | 101.11 | 102.11 | 104.07 | 105.73 | 107.04 | 109.35 | 110.01 | 112.96 | 112.98 | 116.58 |
| 0.740 | 114.49 | 114.62 | 115.62 | 116.06 | 118.52 | 119.69 | 121.43 | 123.30 | 124.34 | 126.92 | 127.25 | 130.54 | 130.17 | 134.16 |
| 0.720 | 134.64 | 135.85 | 135.75 | 136.29 | 138.59 | 139.91 | 141.44 | 143.53 | 144.50 | 147.15 | 147.16 | 150.77 | 150.02 | 154.38 |
| 0.700 | 157.90 | 158.12 | 158.99 | 159.57 | 161.77 | 163.19 | 164.57 | 166.81 | 167.37 | 170.42 | 170.17 | 174.04 | 172.98 | 177.66 |
| 0.680 | 184.84 | 184.90 | 185.90 | 186.35 | 188.63 | 189.17 | 191.37 | 193.59 | 195.11 | 197.21 | 196.86 | 200.82 | 199.61 | 204.44 |
| 0.660 | 216.10 | 215.69 | 217.14 | 217.14 | 219.81 | 220.76 | 222.49 | 224.38 | 225.18 | 227.99 | 227.87 | 231.61 | 230.56 | 235.23 |



Figure 1. Pressure-volume relation of Au and Hg : __, present work; *, experimental Hg data, $0-13 \mathrm{kbar}, 21.9^{\circ} \mathrm{C}$ from [16]; - experimental Hg data, $0-13 \mathrm{kbar}, 40.5^{\circ} \mathrm{C}$, from [16]; O, experimental Hg data, $0-13 \mathrm{kbar}, 52.9^{\circ} \mathrm{C}$, from [16]; $\Delta$, experimental Au data, $0-250 \mathrm{GPa}$, 500 K from [15]; $\odot$, experimental Au data, $0-250 \mathrm{GPa}, 1000 \mathrm{~K}$ from [15]; $\times$, experimental Au data, $0-250 \mathrm{GPa}, 1500 \mathrm{~K}$, from [15].

Therefore, the parameter $Z$ in the present theory is taken to be independent of both the pressure and the temperature. This assumption works very well and it is quite evident from the results reported later in tables 2 and 3 .
(iv) The present EOS can also give the bulk modulus as a function of volume. For this purpose, we differentiate either equation (5) with respect to pressure or equation (8) with respect to volume at constant temperature and the result is

$$
\begin{equation*}
B_{T}(V, T)=\eta /\left\{Z\left[1+\beta\left(\frac{V(P, T)}{V\left(0, T_{R}\right)}\right)^{\eta}\right]\right\} \tag{10}
\end{equation*}
$$

(v) The Anderson-Grüneisen parameter $\delta_{T}(P, T)$ is defined as

$$
\begin{equation*}
\delta_{T}(P, T)=-\frac{1}{\alpha(P, T) B_{T}(P, T)}\left(\frac{\partial B_{T}(P, T)}{\partial T}\right)_{P} \tag{11}
\end{equation*}
$$

Dass and Kumari [11] have obtained this parameter as

$$
\begin{equation*}
\delta_{T}(P, T)=B_{T}^{\prime}(P, T) \tag{12}
\end{equation*}
$$

Table 3. Comparison of density as a function of pressure and temperature for Mo and W.

| $\begin{aligned} & P \\ & (\mathrm{GPa}) \end{aligned}$ | W |  |  | Mo |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $T$ <br> (K) | $\rho\left(\mathrm{g} \mathrm{cm}^{-3}\right)$ |  | $\begin{aligned} & T \\ & (\mathrm{~K}) \end{aligned}$ | $\rho\left(\mathrm{g} \mathrm{cm}^{-3}\right)$ |  |
|  |  | From [14] | Calculated |  | From [14] | Calculated |
| 0.0 | 293.0 | 19.256 | 19.256 | 293.0 | 10.215 | 10.215 |
| 10.0 | 309.0 | 19.846 | 19.841 | 311.0 | 10.577 | 10.573 |
| 20.0 | 332.0 | 20.388 | 20.383 | 338.0 | 10.906 | 10.901 |
| 30.0 | 336.0 | 20.891 | 20.886 | 378.0 | 11.208 | 11.205 |
| 40.0 | 413.0 | 21.360 | 21.358 | 435.0 | 11.489 | 11.487 |
| 50.0 | 477.0 | 21.802 | 21.801 | 510.0 | 11.752 | 11.751 |
| 60.0 | 556.0 | 22.219 | 22.220 | 604.0 | 11.999 | 12.000 |
| 70.0 | 652.0 | 22.614 | 22.618 | 716.0 | 12.233 | 12.235 |
| 80.0 | 765.0 | 22.991 | 22.996 | 847.0 | 12.455 | 12.458 |
| 90.0 | 894.0 | 23.351 | 23.357 | 996.0 | 12.666 | 12.671 |
| 100.0 | 1040.0 | 23.695 | 23.702 | 1163.0 | 12.868 | 12.874 |
| 110.0 | 1202.0 | 24.026 | 24.033 | 1347.0 | 13.062 | 13.068 |
| 120.0 | 1380.0 | 24.345 | 24.352 | 1549.0 | 13.248 | 13.254 |
| 130.0 | 1573.0 | 24.652 | 24.659 | 1767.0 | 13.427 | 13.434 |
| 140.0 | 1582.0 | 24.948 | 24.956 | 2002.0 | 13.599 | 13.607 |
| 150.0 | 2006.0 | 25.235 | 25.243 | 2252.0 | 13.766 | 13.774 |
| 160.0 | 2244.0 | 25.513 | 25.521 | 2517.0 | 13.927 | 13.936 |
| 170.0 | 2496.0 | 25.782 | 25.790 | 2798.0 | 14.083 | 14.093 |
| 180.0 | 2762.0 | 26.044 | 26.052 | 3092.0 | 14.235 | 14.245 |
| 190.0 | 3042.0 | 26.298 | 26.307 | 3401.0 | 14.382 | 14.393 |
| 300.0 | 3334.0 | 26.546 | 26.555 | 3723.0 | 14.525 | 14.537 |
| 210.0 | 3639.0 | 26.787 | 26.797 | 4058.0 | 14.664 | 14.677 |
| 220.0 | 3957.0 | 27.022 | 27.032 | 4406.0 | 14.800 | 14.814 |
| 230.0 | 4286.0 | 27.251 | 27.263 | 4766.0 | 14.932 | 14.947 |
| 240.0 | 4628.0 | 27.475 | 27.488 | 5139.0 | 15.061 | 15.077 |
| 250.0 | 4980.0 | 27.693 | 27.708 | 5523.0 | 15.187 | 15.205 |
| 260.0 | 5344.0 | 27.907 | 27.924 | 5918.0 | 15.310 | 15.329 |
| 270.0 | 5719.0 | 28.116 | 28.135 | 6324.0 | 15.430 | 15.452 |
| 280.0 | 6104.0 | 28.320 | 28.342 | 6741.0 | 15.548 | 15.571 |
| 290.0 | 6499.0 | 28.521 | 28.545 | 7168.0 | 15.663 | 15.689 |
| 300.0 | 6905.0 | 28.717 | 28.744 | 7605.0 | 15.776 | 15.804 |

with the help of equation (9). On the other hand, Chang [13] has given this parameter as

$$
\begin{equation*}
\delta_{T}(P, T)=B_{T}^{\prime}(P, T)-1 \tag{13}
\end{equation*}
$$

The relation given by equation (12) is certainly better than that of equation (13) and has already been discussed elsewhere [4]. Thus, the variation in $\delta_{T}(P, T)$ with pressure and temperature can be computed with the help of equation (7).
(vi) The present EOS can also be used to obtain a number of thermodynamic properties as a function of pressure. This will be discussed elsewhere.

## 3. Comparison with experiment

We shall be able to compare the predictions of equations (5)-(7) with the experimental data provided that the best fitted values of $B_{T}\left(0, T_{R}\right), B_{T}^{\prime}\left(0, T_{T}\right)$ and $Z$ become available besides the value of $\alpha\left(0, T_{R}\right)$. The values of the first three parameters are obtained by the least-squares fitting technique for $\mathrm{Au}, \mathrm{Mo}, \mathrm{W}$ and Hg at a reference temperature $T_{R}$. The


Figure 2. Pressure-volume relation of Mo and W at 1000 K : __, present work; $\times$, W data, from [15], - Mo data, from [15].
values of these parameters together with other relevant data are reported in table 1. The experimental volume data are taken from Heinz and Jeanolz [14] for Au, from Hixson and Fritz [15] for Mo and W and from Davis and Gordon [16] for Hg.

### 3.1. Compression studies

By making use of equation (5) and taking the values of the relevant parameters from table 1, the volume is computed for $\mathrm{Mo}, \mathrm{W}$ and Hg as a function of pressure at different temperatures. The pressure is calculated as a function of volume at different temperatures for Au .

The calculated values of $P$ for Au are compared with the available data of Heinz and Jeanolz in table 1 and are plotted in figure 1. The agreement is very good as the discrepancy lies within $\pm 3.5 \%$ in the total ranges of pressures and temperatures.

The computation of density is done with the help of equation (5) for Mo and W. The results are compared with the Hugoniot results of Hixson and Fritz in table 3. The discrepancy is $\pm 0.2 \%$ for Mo and $\pm 0.15 \%$ for W in the total ranges of pressures and temperatures. Thus, very good agreement is observed here.

Table 4. Comparison of volume and $B_{T}(0, T)$ as functions of pressure and temperature for liquid Hg . The results in parentheses are taken from Davis and Gordon [15].

|  | $T=21.09{ }^{\circ} \mathrm{C}$ |  | $T=40.5{ }^{\circ} \mathrm{C}$ |  | $T=52.9{ }^{\circ} \mathrm{C}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P$ <br> (kbar) | $\begin{aligned} & V(P, T) \\ & \left(V=1 \text { at } 0^{\circ} \mathrm{C}\right) \end{aligned}$ | $B_{T}(P, T)$ <br> (kbar) | $\begin{aligned} & V(P, T) \\ & \left(V=1 \text { at } 0^{\circ} \mathrm{C}\right) \end{aligned}$ | $\overline{B_{T}(P, T)}$ <br> (kbar) | $\begin{aligned} & V(P, T) \\ & \left(V=1 \text { at } 0^{\circ} \mathrm{C}\right) \end{aligned}$ | $B_{T}(P, T)$ <br> (kbar) |
| 0.0 | $\begin{gathered} 1.00398 \\ (1.00398) \end{gathered}$ | $\begin{gathered} 248.4 \\ (248.4) \end{gathered}$ | $\begin{gathered} 1.00740 \\ (1.00736) \end{gathered}$ | $\begin{gathered} 240.7 \\ (243.1) \end{gathered}$ | $\begin{gathered} 1.00974 \\ (1.00962) \end{gathered}$ | $\begin{gathered} 235.5 \\ (239.6) \end{gathered}$ |
| 1.0 | $\begin{gathered} 1.00000 \\ (1.00000) \end{gathered}$ | $\begin{gathered} 257.5 \\ (257.6) \end{gathered}$ | $\begin{gathered} 1.00330 \\ (1.00330) \end{gathered}$ | $\begin{gathered} 249.9 \\ (252.4) \end{gathered}$ | $\begin{gathered} 1.00555 \\ (1.00551) \end{gathered}$ | $\begin{gathered} 244.8 \\ (248.9) \end{gathered}$ |
| 2.0 | $\begin{gathered} 0.99621 \\ (0.99621) \end{gathered}$ | $\begin{gathered} 266.5 \\ (266.6) \end{gathered}$ | $\begin{gathered} 0.999366 \\ (0.999410) \end{gathered}$ | $\begin{gathered} 259.0 \\ (261.3) \end{gathered}$ | $\begin{gathered} 1.00153 \\ (1.00156) \end{gathered}$ | $\begin{gathered} 253.9 \\ (257.9) \end{gathered}$ |
| 3.0 | $\begin{gathered} 0.992539 \\ (0.992540) \end{gathered}$ | $\begin{gathered} 275.3 \\ (275.3) \end{gathered}$ | $\begin{gathered} 0.995581 \\ (0.995650) \end{gathered}$ | $\begin{gathered} 267.9 \\ (270.1) \end{gathered}$ | $\begin{gathered} 0.997661 \\ (0.99774) \end{gathered}$ | $\begin{gathered} 262.9 \\ (266.7) \end{gathered}$ |
| 4.0 | $\begin{gathered} 0.988996 \\ (0.98000) \end{gathered}$ | $\begin{gathered} 283.9 \\ (284.0) \end{gathered}$ | $\begin{gathered} 0.991932 \\ (0.992030) \end{gathered}$ | $\begin{gathered} 276.7 \\ (278.7) \end{gathered}$ | $\begin{gathered} 0.993938 \\ (0.994060) \end{gathered}$ | $\begin{gathered} 271.8 \\ (275.4) \end{gathered}$ |
| 5.0 | $\begin{gathered} 0.985571 \\ (0.985570) \end{gathered}$ | $\begin{gathered} 292.5 \\ (292.5) \end{gathered}$ | $\begin{gathered} 0.988410 \\ (0.988530) \end{gathered}$ | $\begin{gathered} 285.3 \\ (287.3) \end{gathered}$ | $\begin{gathered} 0.990347 \\ (0.990510) \end{gathered}$ | $\begin{gathered} 280.6 \\ (283.9) \end{gathered}$ |
| 6.0 | $\begin{gathered} 0.982255 \\ (0.982250) \end{gathered}$ | $\begin{gathered} 300.9 \\ (300.8) \end{gathered}$ | $\begin{gathered} 0.985004 \\ (0.985140) \end{gathered}$ | $\begin{gathered} 293.9 \\ (295.6) \end{gathered}$ | $\begin{gathered} 0.986878 \\ (0.997070) \end{gathered}$ | $\begin{gathered} 289.1 \\ (292.3) \end{gathered}$ |
| 7.0 | $\begin{gathered} 0.979041 \\ (0.979040) \end{gathered}$ | $\begin{gathered} 309.2 \\ (309.1) \end{gathered}$ | $\begin{gathered} 0.981706 \\ (0.981860) \end{gathered}$ | $\begin{gathered} 302.3 \\ (303.9) \end{gathered}$ | $\begin{gathered} 0.983521 \\ (0.983730) \end{gathered}$ | $\begin{gathered} 297.6 \\ (300.5) \end{gathered}$ |
| 8.0 | $\begin{gathered} 0.975921 \\ (0.975900) \end{gathered}$ | $\begin{gathered} 317.4 \\ (317.0) \end{gathered}$ | $\begin{gathered} 0.978508 \\ (0.978600) \end{gathered}$ | $\begin{gathered} 310.6 \\ (312.0) \end{gathered}$ | $\begin{gathered} 0.980268 \\ (0.980510) \end{gathered}$ | $\begin{gathered} 306.0 \\ (309.0) \end{gathered}$ |
| 9.0 | $\begin{gathered} 0.972889 \\ (0.972900) \end{gathered}$ | $\begin{gathered} 325.4 \\ (325.0) \end{gathered}$ | $\begin{gathered} 0.975403 \\ (0.975600) \end{gathered}$ | $\begin{gathered} 318.7 \\ (320.0) \end{gathered}$ | $\begin{gathered} 0.977113 \\ (0.977380) \end{gathered}$ | $\begin{gathered} 314.2 \\ (317.0) \end{gathered}$ |
| 10.0 | $\begin{gathered} 0.96940 \\ (0.969900) \end{gathered}$ | $\begin{gathered} 333.3 \\ (333.0) \end{gathered}$ | $\begin{gathered} 0.972386 \\ (0.972600) \end{gathered}$ | $\begin{gathered} 326.7 \\ (328.0) \end{gathered}$ | $\begin{gathered} 0.974048 \\ (0.974340) \end{gathered}$ | $\begin{gathered} 322.2 \\ (325.0) \end{gathered}$ |
| 11.0 | $\begin{gathered} 0.967067 \\ (0.967100) \end{gathered}$ | $\begin{gathered} 341.1 \\ (341.0) \end{gathered}$ | $\begin{gathered} 0.969450 \\ (0.969700) \end{gathered}$ | $\begin{gathered} 334.6 \\ (336.0) \end{gathered}$ | $\begin{gathered} 0.971067 \\ (0.971380) \end{gathered}$ | $\begin{gathered} 330.2 \\ (333.0) \end{gathered}$ |
| 12.0 | $\begin{gathered} 0.964267 \\ (0.964300) \end{gathered}$ | $\begin{gathered} 348.7 \\ (349.0) \end{gathered}$ | $\begin{gathered} 0.966590 \\ (0.966800) \end{gathered}$ | $\begin{gathered} 342.3 \\ (344.0) \end{gathered}$ | $\begin{gathered} 0.968166 \\ (0.968510) \end{gathered}$ | $\begin{gathered} 338.0 \\ (341.0) \end{gathered}$ |
| 13.0 | $\begin{gathered} 0.961536 \\ (0.961500) \end{gathered}$ | $\begin{gathered} 356.3 \\ (357.0) \end{gathered}$ | $\begin{gathered} 0.963802 \\ (0.964000) \end{gathered}$ | $\begin{gathered} 349.9 \\ (352.0) \end{gathered}$ | $\begin{gathered} 0.965338 \\ (0.965720) \end{gathered}$ | $\begin{gathered} 345.7 \\ (348.0) \end{gathered}$ |

It is worthwhile to mention here that it is not possible to plot the data in table 3 because both $P$ and $T$ are varying. However, $1-V(P, T) / V\left(0, T_{R}\right)$ is plotted against $P$ for Mo and W at 1000 K in figure 2 and the agreement is very good. The same is also true at other temperatures too.

The relative volume calculations are done for liquid Hg at $21.9,40.5$ and $52.9^{\circ} \mathrm{C}$ up to 13 kbar pressure. The calculated results are compared with the data of Davis and Gordon in table 4 and are plotted in figure 1 ; they agree within $\pm 0.05 \%$ in the total ranges of pressures and temperatures.

Thus, it can be said from the above discussion that the present EOS is quite successful in representing the volume data as a function of pressure at different temperatures. This gives confidence in the use of the present EOS for high pressures and high temperatures.

### 3.2. Bulk modulus and its first derivative

The isothermal bulk modulus and its first pressure derivative as functions of pressure at different temperatures can be obtained from equations (6) and (7), respectively, after taking the values of the relevant parameters from table 1.


Figure 3. Variation in $B_{T}(P, T)$ with $P$ for Hg and with temperature for $\mathrm{Au}: \quad \_$, present work; •, experimental Hg data, $0-13 \mathrm{kbar}, 21.9^{\circ} \mathrm{C}$, from [16]; $\times$, experimental Hg data, $0-$ $13 \mathrm{kbar}, 52.9^{\circ} \mathrm{C}$, from [16]; - , experimental Au data, $P=0,0-550 \mathrm{~K}$, from [17].

Calculations for $B_{T}(P, T)$ as a function of pressure at different temperatures are carried out for Hg . The calculated results are compared with the data of Davis and Gordon in table 4 and are plotted in figure 3. The agreement is very good.

For Au , equation (6) is used to compute the temperature dependence of the bulk modulus at $P=0$. The calculated results are compared with the data given by Anderson et al [17] in table 5 and are plotted in figure 3. The agreement is fairly good. However, our results have a small temperature dependence.

No data are available for the bulk modulus as a function of pressure for $\mathrm{Au}, \mathrm{Mo}$ and W. However, the calculations are done within the $0-400$ kbar pressure range for each solid at 300 K with the help of equation (6) after making use of the relevant parameters from table 1. The results so obtained are plotted in figure 4. Furthermore, for completeness, we also report the temperature dependence of the bulk modulus for Mo and W in table 5 for ready reference.

As far as the calculations of $B_{T}^{\prime}(P, T)$ are concerned, no experimental or theoretical data are available for the materials studied here and hence no comparison is made. However, any one who is interested in this parameter can easily obtain the value at the desired pressure and temperature.

Table 5. Calculation of isothermal bulk modulus as a function of temperature for $\mathrm{Au}, \mathrm{Mo}$ and W .

|  | Au |  |  |  | Mo |
| ---: | :--- | :--- | :--- | :--- | :--- |
| $T$ | $B_{T}(0, T)$, <br> $[17]$ | $B_{T}(0, T)$, <br> calculated <br> $(\mathrm{GPa})$ | $T$ <br> $(\mathrm{~K})$ | W <br> $B_{T}(0, T)$, <br> calculated <br> $(\mathrm{GPa})$ | $B_{T}(0, T)$, <br> calculated <br> $(\mathrm{GPa})$ |
| 0.0 | 180.32 | 180.07 | 25.0 | 274.80 | 317.98 |
| 10.0 | 180.32 | 179.71 | 50.0 | 274.43 | 317.59 |
| 25.0 | 180.13 | 179.17 | 75.0 | 274.07 | 317.20 |
| 50.0 | 179.33 | 178.26 | 100 | 273.00 | 316.81 |
| 75.0 | 178.09 | 177.35 | 150 | 272.98 | 316.09 |
| 100.0 | 176.73 | 176.44 | 200 | 272.25 | 315.25 |
| 125.0 | 175.41 | 175.53 | 300 | 270.80 | 313.69 |
| 150.0 | 174.07 | 174.62 | 400 | 269.34 | 312.13 |
| 175.0 | 172.81 | 173.71 | 500 | 267.89 | 310.57 |
| 200.0 | 171.53 | 172.80 | 600 | 266.43 | 309.01 |
| 225.0 | 170.24 | 171.88 | 700 | 264.97 | 307.45 |
| 250.0 | 168.99 | 170.98 | 800 | 263.52 | 305.89 |
| 275.0 | 167.76 | 170.06 | 900 | 262.06 | 304.33 |
| 300.0 | 166.51 | 169.15 | 1000 | 260.60 | 302.76 |
| 350.0 | 163.94 | 167.33 | 1200 | 257.68 | 299.64 |
| 400.0 | 161.22 | 165.50 |  |  |  |
| 450.0 | 158.91 | 163.67 |  |  |  |
| 500.0 | 156.49 | 161.84 |  |  |  |
| 550.0 | 153.71 | 160.02 |  |  |  |

Table 6. Comparison of $\alpha(0, T)\left(\times 10^{-6} \mathrm{~K}^{-1}\right)$ as a function of temperature for Au , Mo and W ; $\alpha_{\text {exp }}(0, T)$ are the experimental values taken from [18]; $\alpha_{\text {calc }}$, our calculated values.

| $T$ <br> (K) | Mo |  | W |  | Au |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\alpha_{\text {exp }}(0, T)$ | $\alpha_{\text {calc }}(0, T)$ | $\alpha_{\text {exp }}(0, T)$ | $\alpha_{\text {calc }}(0, T)$ | $\alpha_{\text {exp }}(0, T)$ | $\alpha_{\text {calc }}(0, T)$ |
| 100 | - | - | - | - | 35.7 | 41.01 |
| 150 | 12.0 | 13.93 | 10.8 | 12.61 | 39.3 | 41.44 |
| 200 | 13.8 | 14.29 | 12.3 | 12.90 | 40.8 | 41.88 |
| 250 | 14.7 | 14.66 | 13.2 | 13.22 | 42.0 | 42.32 |
| 293 | 15.0 | 15.00 | 13.5 | 13.50 | 42.6 | 42.72 |
| 350 | 15.3 | 15.47 | 13.8 | 13.89 | 44.1 | 43.72 |
| 400 | 15.6 | 15.92 | 13.8 | 14.26 | 45.6 | 44.71 |
| 500 | 15.6 | 16.88 | 13.8 | 15.05 | 47.4 | 45.75 |
| 600 | 16.2 | 17.98 | 14.1 | 15.95 | 49.2 | 46.83 |
| 700 | 16.5 | 19.23 | 14.4 | 16.95 | 51.3 | 47.97 |
| 800 | 17.1 | 20.67 | 14.4 | 18.10 | 56.4 | 49.17 |
| 900 | 18.3 | 22.36 | 14.4 | 19.42 | 63.3 | 50.43 |

### 3.3. Thermal expansion

The calculations of $\alpha(0, T)$ are done with the help of equation (9) for $\mathrm{Au}, \mathrm{Mo}$ and W . The results are compared with the experimental data [18] for each solid in table 6 . The overall agreement is good. However, the discrepancy increases with a rise in temperature.


Figure 4. Variation in $B_{T}(P, T)$ with $P$ for Au , Mo and W at 300 K : - , present work.

### 3.4. The Anderson-Grüneisen parameter

According to $[4,11]$, the Anderson-Grüneisen parameter is given by

$$
\delta_{T}(P, T)=B_{T}^{\prime}(P, T)
$$

Thus, the variation in $\delta_{T}(P, T)$ with pressure and temperature is the same as that of $B_{T}^{\prime}(P, T)$, i.e. $\delta_{T}(P, T)$ increases with rising temperature and decreases with increasing pressure.

## 4. Summary

The isothermal bulk modulus and its first pressure derivative, thermal expansion coefficient and the Anderson-Grüneisen parameter can be computed simultaneously together with volume compression as functions of pressure at different temperatures. The overall agreement between the calculated and the experimental value for various properties of $\mathrm{Au}, \mathrm{Mo}, \mathrm{W}$ and Hg is good. Thus, it appears that the present temperature-dependent EOS has great potential and scope as far as the practical applications in condensed matter are concerned.

## Acknowledgments

One of the authors (PK) is grateful to the University Grants Commission, New Delhi, for the award of Junior Research Fellowship. Another author (RK) is grateful to the Physics Department, University of Roorkee, Roorkee, for providing him with the facilities to carry out the research work.

## References

[1] Murnagham F D 1951 Finite Deformation of an Elastic Solid (New York: Wiley) ch 4
[2] Birch F 1952 J. Geophys. Res. 57227
[3] Vinet P, Ferrante J, Smith J R and Ross J H 1986 J. Phys. C: Solid State Phys. 19 L467
[4] Kumari M and Dass N 1990 J. Phys.: Condens. Matter 23219
[5] Vinet P, Smith J R, Farrante J and Rose J H 1987 Phys. Rev. B 351945
[6] Parsafar G and Mason E A 1994 Phys. Rev. B 193049
[7] Kumari M and Dass N 1990 J. Phys.: Condens. Matter 27891
[8] Kumari M and Dass N 1991 J. Phys.: Condens. Matter 34099
[9] Anderson O L 1979 J. Geophys. Res. 843537
[10] Kumari M and Dass N 1993 J. Non-Cryst. Solids 156-8 417
[11] Dass N and Kumari M 1984 Phys. Status Solidi b 124531
[12] Kumari M and Dass N 1986 Phys. Status Solidi b 133101
[13] Chang Y A 1967 J. Phys. Chem. Solids 28697
[14] Heinz D L and Jeanloz R 1984 J. Appl. Phys. 55885
[15] Hixson R S and Fritz J N 1991 J. Appl. Phys. 711721
[16] Davis L A and Gordon R B 1967 J. Chem. Phys. 462650
[17] Anderson O L, Issak D G and Yamamoto S 1989 J. Appl. Phys. 651534
[18] Kennedy G C and Keller R N 1972 American Institute of Physics Handbook 3rd edn, ed D E Gray (New York: McGraw-Hill) pp 4-38

